Calculating the Mass and Density of Earth

Kepler's Third Law (Newton's Form):

$$\frac{P^2}{a^3} = \frac{4\pi^2}{MG}$$

The period of the moon's orbit is 27.32 days, or 2,361,000 seconds (roughly). The semi-major axis (*a*) is 384,400,000 m. *G*, the gravitational constant, is presently calculated to be $6.673 \cdot 10^{-11}$ $\mathrm{m^3 kg^{-1} s^{-2}}$. Doing the math:

$$\frac{P^2}{a^3} = \frac{4\pi^2}{MG} \quad \Rightarrow \quad M = \frac{4\pi^2 a^3}{GP^2} = \frac{4\pi^2\left(3.844 \cdot 10^8 \text{ m}\right)^3}{\left(6.673 \cdot 10^{-11} \ \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}\right)\left(2.361 \cdot 10^6 \text{ s}\right)^2} \approx 6 \cdot 10^{24} \text{ kg}$$

The value from NASA is $5.9736 \cdot 10^{24}$ kg.

Taking an average radius of Earth of 6371 km, the density can be calculated:

$$\rho = \frac{M}{V} = \frac{6 \cdot 10^{24} \text{ kg}}{\frac{4}{3}\pi r^3} = \frac{6 \cdot 10^{27} \text{ g}}{\frac{4}{3}\pi\left(6.371 \cdot 10^8 \text{ cm}\right)^3} \approx 5.54 \text{ g} \cdot \text{cm}^{-3}$$

The density for a spherical shell 300 km thick or 1400 km thick:

$$\rho = \frac{M}{V} = \frac{6 \cdot 10^{24} \text{ kg}}{\frac{4}{3}\pi\left(r_o - r_i\right)^3} = \frac{6 \cdot 10^{27} \text{ g}}{\frac{4}{3}\pi\left(\left(6.371 \cdot 10^8 \text{ cm}\right)^3 - \left(6.071 \cdot 10^8 \text{ cm}\right)^3\right)} \approx 40.9 \text{ g} \cdot \text{cm}^{-3}$$

$$\rho = \frac{M}{V} = \frac{6 \cdot 10^{24} \text{ kg}}{\frac{4}{3}\pi\left(r_o - r_i\right)^3} = \frac{6 \cdot 10^{27} \text{ g}}{\frac{4}{3}\pi\left(\left(6.371 \cdot 10^8 \text{ cm}\right)^3 - \left(4.971 \cdot 10^8 \text{ cm}\right)^3\right)} \approx 10.5 \text{ g} \cdot \text{cm}^{-3}$$